Spectral characterization of the Hill's equation related to different boundary conditions

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Talk Abstract

The subject of nonlinear boundary value problems has been widely considered for a long time in the literature, with special attention to the existence and multiplicity of solutions for such problems. In order to construct an equivalent integral operator, whose fixed points coincide with the solutions of the studied differential problem, it is necessary to describe the spectrum of the related linear part of the equation. This is due to the fact that topological methods, such as degree theory or fixed point index in cones, coupled to iterative techniques and lower and upper solutions methods, are mainly based on the construction of such integral operator. To ensure the existence of this integral operator it is necessary to study carefully the eigenvalues of the linear part of the studied equation. These eigenvalues also appear in the search of constant sign solutions as they usually define the limits of the regions in which the corresponding Green's functions have negative or positive sign on its square of definition. This constant sign for the Green's function related to the considered linear problem is an usual hypothesis when looking for positive solutions of the problem or when monotone iterative techniques are used (see [1, 4] and references therein)

In this talk we will characterize the spectrum of the second order Hill's equation coupled to several boundary value conditions. In particular, we will study the spectrum of the second-order differential Hill's equation coupled to Neumann, Dirichlet, Periodic and Mixed boundary conditions, by applying a linking equality proved by the authors in [2] and expressing the Green's function of the Hill's equation coupled to a given boundary condition as a combination of the Green's function related to another different boundary condition. These spectra are characterized as suitable sets of real values that verify an equality that depends on the Green's function of each case. We will also deduce some properties of these spectra and identities between Green's functions related to different boundary conditions.

As an application of the obtained characterizations, we deduce the existence of solutions of suitable nonlinear boundary value problems.

The obtained results are published in [3].

Keywords: Green's function, Two point boundary conditions, Spectral theory, Comparison results.

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References

[1] A. Cabada, *Green's Functions in the Theory of Ordinary Differential Equations*, Springer Briefs in Mathematics, Springer, New York, NY (2014).

[2] A. Cabada, L. López-Somoza, M. Yousfi, Existence of solutions of nonlinear systems subject to arbitrary linear non-local boundary conditions, *J. Fixed Point Theory Appl.* 25 (2023), no. 4, Paper No. 81, 24 pp.

[3] A. Cabada, L. López-Somoza, M. Yousfi. Characterization of the spectra of the Hill's equation coupled to different boundary value conditions and application to nonlinear boundary problems. *Filomat* 38 (2024), 1, 195–215.

[4] G. S. Ladde, V. Lakshmikantham, A. S. Vatsala, *Monotone iterative techniques for nonlinear differential equations.* Monographs, Advanced Texts and Surveys in Pure and Applied Mathematics, 27. Pitman (Advanced Publishing Program), Boston, MA; distributed by John Wiley & Sons, Inc., New York, 1985.