Harvesting populations in random environments: Shortcomings of the optimal harvesting policy, proposal of alternatives, and their assessment

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Talk Abstract

In a randomly varying environment, the dynamics of a harvested (say, a fish) population with size $X(t)$ can be described by a stochastic differential equations – SDE (see, for instance, [1, 2]). Let the harvesting profit per unit time (p.u.t.) $\Pi(t)$ be the difference between the selling price p.u.t. of the harvest (assumed proportional to the harvesting yield $H(t) = qE(t)X(t)$, where q is the catchability and $E(t)$ the harvesting effort) and the costs p.u.t. of harvesting (assumed to be a quadratic function of $E(t)$). Using stochastic control theory, one can determine the optimal harvesting policy $E^*(t)$ ($0 \leq t \leq T$), i.e. the policy $E(t)$ that maximizes the expected discounted profit $V = \mathbb{E}\left[\int_0^T e^{-\delta t}\Pi(t)dt\right]$, where $\delta > 0$ is a discount rate.

Illustrating with the logistic growth model and parameters provided in [5] for the Pacific halibut, we show that, due to the fast and abrupt random variations in the harvesting effort associated with the environmental induced population size fluctuations, such an optimal policy is incompatible with the logistics of fisheries and therefore inapplicable. It also causes social problems, such as fishermen's unemployment during periods of no or low harvesting. Furthermore, it requires knowledge of the population size at each instant, and estimating population size is an inaccurate, lengthy, and expensive task.

To overcome some or all of these shortcomings, we have proposed alternative sub-optimal harvesting policies and assessed them in terms of applicability, possible social problems, and profit (see [2, 3, 4]). Among the alternatives, we consider:

(a) *constant effort policies*, in which $E(t) \equiv E$;

(b) *stepwise effort policies*, in which $E(t)$ is determined at the beginning of each year (or biennium) according to optimal control theory and kept constant throughout that year (or biennium);

(c) *penalized effort policies*, which incorporate a running energy artificial cost based on deviations of the effort from a reference value in order to avoid abrupt effort changes and periods of low or no fishing;

(d) *stepwise penalized effort policies*, which apply the stepwise procedure to penalized effort policies.

Some of these are nearly optimal and simple to implement in practice.

We have also studied the case of populations with Allee effects (see [3]), not reported here due to time constraints.

Keywords: harvesting models, stochastic differential equations, logistic growth, sub-optimal harvesting policies.

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