Numerical simulations using mass-diffusive compressible fluids flows models

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Talk Abstract

This contribution presents first numerical tests of some recently published alternative models for solution of viscous compressible and nearly incompressible models. All models are solved by high resolution compact finite difference scheme with strong stability preserving Runge-Kutta time stepping. Three simple but challenging computational test cases are presented, based on the double-periodic shear layer, Taylor-Green vortex and the Kelvin-Helmholtz instability. The obtained time-dependent flow fields are showing pronounced shear and vorticity layers being resolved by the standard as well as by the new mass-diffusive modified models. The preliminary results show that in some cases the new models are a viable alternative to the well established classical models. Theoretical analysis and numerical solution of various fluids flows problems poses a challenging problem. The widely used mathematical models describing the compressible fluids flows and incompressible fluids flows are the Navier-Stokes-Fourier and the incompressible Navier Stokes systems respectively. These are mixed type systems of non-linear strongly coupled partial differential equations of hyperbolic, parabolic and elliptic type. Their mathematical analysis as well as numerical solution remains one of the most difficult problems of contemporary science. Recently there have been attempts to revise and possibly improve the traditional mathematical models describing the fluids flows. The works of Brennen [2] and Svärd [6] are example of such possible model updates.In these new models, the basic physical principles (conservation/balance laws) are still being used, but the interpretation of certain physical variables and processes brings other options for into the mathematical formulations of such revised models. These changes are bringing some interesting results from the point of view of mathematical analysis [4] of the corresponding models as well as possible increase in the efficiency of numerical methods [3], [5]. The aim of this paper is to present the initial results of a computational study based on the mass-diffusive compressible and nearly-incompressible fluids flows models based on the works of Svärd [6] extended by Kajzer & Pozorski

in [5]. The new alternative models are first presented, side by side with the standard systems for both compressible and incompressible fluids flows. The new, mass-diffusive models are then solved by high-resolution compact finite-difference methods. The model results are mutually compared for two test cases, documenting the agreement and comparative advantages of the newly formulated models. The full system of *Navier-Stokes-Fourier (NSF)* equations describing the flow of a compressible heat conducting fluid can be written as

$$\begin{aligned}
\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) &= 0 && (1)\\
\partial_t (\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v} + p\mathbb{I}) &= \boldsymbol{\nabla} \cdot \mathbb{S} && (2)\\
\partial_t E + \boldsymbol{\nabla} \cdot \big((E+p)\boldsymbol{v}\big) &= \boldsymbol{\nabla} \cdot \big(\mathbb{S} \cdot \boldsymbol{v} + \kappa \boldsymbol{\nabla} T\big) && (3)
\end{aligned}$$

Here the $\boldsymbol{v}$ is the fluid velocity, ρ density, p pressure. The fluid total energy E is defined as

$$E = \frac{1}{2}\rho|\boldsymbol{v}|^2 + \frac{p}{\gamma - 1} \qquad \text{where} \qquad \gamma = \frac{c_p}{c_v} \tag{4}$$

for the perfect gas obeying the state equation $p = \rho\, R\, T$ with the gas constant $R = c_p - c_v$ being linked to heat capacities c_p, c_v at constant pressure and volume respectively. The stress tensor $\mathbb{S}$ for Newtonian fluid is then defined as

$$\mathbb{S} = \mu \left(\boldsymbol{\nabla} \boldsymbol{v} + \boldsymbol{\nabla} \boldsymbol{v}^T - \frac{2}{3}(\boldsymbol{\nabla} \cdot \boldsymbol{v})\mathbb{I} \right) . \tag{5}$$

The dynamic viscosity μ and heat conductivity κ depend on the fluid considered. The standard *NSF* system (1)–(3) was reformulated by Svärd [6] who replaced it by a *modified Navier-Stokes-Fourier (M-NSF)* system, having similar form, but different right-hand sides in all equations.

$$\begin{aligned}
\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) &= \boldsymbol{\nabla} \cdot \big(\nu\, \boldsymbol{\nabla} \rho\big) && (6)\\
\partial_t (\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v} + p\mathbb{I}) &= \boldsymbol{\nabla} \cdot \big(\nu\, \boldsymbol{\nabla} (\rho \boldsymbol{v})\big) && (7)\\
\partial_t E + \boldsymbol{\nabla} \cdot \big((E+p)\boldsymbol{v}\big) &= \boldsymbol{\nabla} \cdot \big(\nu\, \boldsymbol{\nabla} E\big) && (8)
\end{aligned}$$

The most notable change, probably, is the added mass-diffusive term in the equation for density (6). But the right-hand sides in the momentum and energy equations (7) and (8) have changed as well, consisting now just from the divergence of the gradient of the conserved quantity (in the same form as in the modified mass conservation (6)). The diffusion coefficient $\nu = \mu/\rho$ has now the same value in all the considered equations of the modified *M-NSF* system. The numerical simulations of the alternative compressible as well as nearly incompressible flows models shown their potential in solving problems of practical interest. Although the presented new models offer certain advantages over their classical counterparts (better analytical properties, easier and more efficient numerical implementation), there are numerous issues to be addressed. One of the possible troubles may come from the formulation of the (stress tensor on the) right hand side of the mass-diffusive *M-NSF* model (7). The principle of material frame indifference and the conservation of moment of momentum require the stress tensor to be symmetric (depending just on the symmetric part of velocity gradient). This and many other properties will be in the focus of our future investigation.

Keywords: compressible Navier-Stokes, nearly incompressible flow, mass diffusion, compact finite-difference.

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