Variationals Inequalities for Obstacle Problems with General Growth

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Talk Abstract

Euler-Lagrange equations play a foundamental role when studing the Sobolev regularity of solutions to variational problems such as

$$\min_{v \in K} \int_\Omega F(x, \nabla v) dx,$$

with $\Omega \subset \mathbb{R}^n$ open and bounded, and K subset of some appropriate Sobolev space. It is well known that when the Lagrangian function satisfies p-growth conditions or (p, q)-growth conditions then it is possible to prove that the solutions satisfy a related Euler-Lagrange equation. In the case of the obstacle problem, due to the constraint on the solutions, it is only possible to write a related variational inequality. In this talk I show how to prove the aforementioned variational inequality when working with more general growths. This study was inspired by [1] and [2]. For our work [3] we assume only superlinear growth for the Lagrangian function F.

Keywords: Variational inequalities, minimizers, obstacle problems, convex extension, monotonic approximation.

References

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