Convertibility of singular symmetric matrices

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Talk Abstract

In this work, a relation between the determinant and the permanent on singular symmetric matrices is established. Several no go theorems have been proved, and some of them related with the conversion of the determinant into the permanent for symmetric matrices and singular matrices separately. The aim of this paper is to reinforce the lack of solution to the convertibility problem concerning to singular symmetric matrices, more concretely, being $H_n(\mathbb{F})$ the linear space of n-square symmetric matrices over a field $\mathbb{F}$ with at least n elements and whose characteristic is not 2, we show that if $n \geq 3$ there is no linear transformation

$$T : H_n(\mathbb{F}) \to H_n(\mathbb{F})$$

satisfying the condition $\det(X) = 0$ if and only if $\mathrm{per}(T(X)) = 0$, for all $X \in H_n(\mathbb{F})$.

Keywords: linear preserver problems, permanent, determinant, symmetric matrices.

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