A comprehensive model for computation and measurement numbers

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Talk Abstract

Historically, a distinction has been made between numbers serving for computations, which are intrinsically mathematic, and numbers serving for measurements [3] of, say, physical quantities. Attempts for modelling have been made by among others Laugwitz [4] and Spalt [6]. Here we aim at an interpretation of computation numbers and measurement numbers using an elementary extension of Nelson's axiomatics as presented in [5]. This nonstandard axiomatics assigns the predicat "standard" only to natural numbers, and concerns a weak fragment of Nonstandard Analysis; Nelson argues that his axiomatics is sufficient to develop advanced stochastics avoiding measure theory, in some exceptional cases he uses a "star" axiom, which corresponds to sequential saturation. The axiomatics permits to define standard integers, rational numbers and algebraic numbers, which could serve as a model for numbers for concrete computations. However it is not possible to incorporate standard real numbers in this approach. We show that the "star" axiom and an elementary fragment of Kanovei and Reeken's axiomatics for external sets HST enable us to define standard real numbers as equivalence classes. Its elements differ at most infinitesimally, and this imprecision makes them a model for measurement numbers. The approach bears some relation with the work on external numbers in [2] and flexible meadows in [1].

Keywords: Computational numbers, measurement numbers, infinitesimals.

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