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Suporte

Electrónico https://www.2iwmps24.uevora.pt/book-of-abstracts/ ISBN: 978-972-778-400-4 Welcome to the 2nd Int. Workshop on Mathematics and Physical Sciences

Dear Participants, Colleagues and Friends

It is a great honor and a privilege to give you all the warmest welcome to the 2nd International Workshop on Mathematics and Physical Sciences (MatPhys).

This conference is being held in the beautiful city of Évora, Portugal. The host institution and the associated scientific research centers are committed to the event, hoping it will be a benchmark for scientific collaboration between experts in Mathematics, Physics, Computer Sciences, and other related fields.

The main purposes are: to discuss and present new trends and significant challenges in mathematics, physical and computer sciences, and their applications; to follow and discuss new topics in the area; to amplify previous and new collaborations between different institutions and areas.

Be welcome,

Feliz Minhós Chair of the conference iv

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List of participants

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	Thursday 11.07.2024	4
08:45h-09:00h 09:00h-09:30h 09:30h-10:15h 10:15h-11:00h	Registration – C Opening session – Alberto Cabada, Chair: Feliz Minhós Sofia Andringa, Chair: Mourad Bezzeghoud	Registration – CES Amphitheatre.131 Opening session – CES Amphitheatre.131 liz Minhós urad Bezzeghoud
11:00h-11:30h	Coffee break	Coffee break – Room CES.124
11:30h–12:05h 12:05h–12:40h	CES Amphitheatre.1 Carlos Braumann Elvira Zappale (via Zoom)	CES Amphitheatre.131 (Chair: Célia Nunes)
12:40h–14:00h	Lunch – CES ((Lunch – CES (Cozinha do Cardeal)
14.00h_14.25h	Room CES.110 (Chair: Luís Silva)	Room CES.115 (Chair: Carlos Braumann)
14.25h-14:50h 14:50h-15:15h	Andrea Torricelli And I. Santos	Luís M. Grilo Célia Nunes
15:15h–15:40h 15:40h–16:05h	José A. Rodrigues Ana C. Carapito	Carla Santos Lígia Ferreira
16:05h-16:30h	Coffee break	Coffee break – Room CES.124
	Room CES.110 (Chair: Luís Bandeira)	Room CES.115 (Chair: Rui J. Oliveira)
16:30h-16:55h	Bruno Dinis	Md T. Ahmed
17:20h-17:45h	limme van den berg João Dias	Nada El Bouziani Hajar Sdira
17:45h-18:10h 18·10h-18·35h	Mouhaydine Tlemçani Mel Surui Ali	Mian Fareed
20:00h		Dinner – CES (Cozinha do Cardeal)

Registration - CES CES Amphith Ces Amphith Pedro Lima, Chair: Fernando Carapau Tomáš Bodnár (via Zoom), Chair: Paulo Correia Luís Silva, Chair: Paulo Correia Luís Silva, Chair: Paulo Correia Coffee break - R Romática Coffee break - R Ashwin Vaidya (via Zoom) Ashwin Vaidya (via Zoom) Joshua Cellar (via Zoom) Joshua Cellar (via Zoom) Joshua Cellar (via Zoom) Alberto Simões Ilda Inácio Anabela Silva Rui J. Oliveira Room CES.110 (Chair: António Anjos) Mouhaydine Tlemçani Souhila Chabane		Friday 12.07.2024	
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	16:30h-16:55h	Mouhaydine Tlemçani	Rui Gomes
	uuz:/1-ucc:01	Sounila Chabane	Jose Borges
	17:20h-17:45h	Conference clos	ing – Room CES.110

Main speakers

Spectral characterization of the Hill's equation related to different boundary conditions

Alberto Cabada^{1,2}, Lucía López-Somoza^{1,2} and Mouhcine Yousfi^{1,2}

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Talk Abstract

The subject of nonlinear boundary value problems has been widely considered for a long time in the literature, with special attention to the existence and multiplicity of solutions for such problems. In order to construct an equivalent integral operator, whose fixed points coincide with the solutions of the studied differential problem, it is necessary to describe the spectrum of the related linear part of the equation. This is due to the fact that topological methods, such as degree theory or fixed point index in cones, coupled to iterative techniques and lower and upper solutions methods, are mainly based on the construction of such integral operator. To ensure the existence of this integral operator it is necessary to study carefully the eigenvalues of the linear part of the studied equation. These eigenvalues also appear in the search of constant sign solutions as they usually define the limits of the regions in which the corresponding Green's functions have negative or positive sign on its square of definition. This constant sign for the Green's function related to the considered linear problem is an usual hypothesis when looking for positive solutions of the problem or when monotone iterative techniques are used (see [1,4] and references therein). In this talk we will characterize the spectrum of the second order Hill's equation coupled to several boundary value conditions. In particular, we will study the spectrum of the second-order differential Hill's equation coupled to Neumann, Dirichlet, Periodic and Mixed boundary conditions, by applying a linking equality proved by the authors in [2] and expressing the Green's function of the Hill's equation coupled to a given boundary condition as a combination of the Green's function related to another different boundary condition. These spectra are characterized as suitable sets of real values that verify an equality that depends on the Green's function of each case. We will also deduce some properties of these spectra and identities between Green's functions related to different boundary conditions. As an application of the obtained characterizations, we deduce the existence of solutions of suitable nonlinear boundary value problems. The obtained results are published in [3].

Keywords: Green's function, Two point boundary conditions, Spectral theory, Comparison results.

Acknowledgements

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Numerical Approximation Of Two-Dimensional Stochastic Neural Field Equations With Finite Transmission Speed

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Talk Abstract

Neural field equations are intended to model the synaptic interactions between neurons in a continuous neural network, called a neural field [1], [2]. This kind of integro-differential equations proved to be a useful tool for the spatiotemporal modeling of the neuronal activity from a macroscopic point of view, allowing the study of a wide variety of neurobiological phenomena, such as the processing of sensory stimuli. The aim of the present talk is to study the effects of additive noise in one- and two-dimensional neural fields, while taking into account finite signal transmission speed. A Galerkin-type method to approximate such models is presented, which applies the Fast Fourier Transformation to optimise the computational effort required to solve this type of equations. Numerical simulations obtained by this algorithm are presented and discussed.

Keywords: Neural field equation, stochastic differential equation, Galerkin method.

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New elementary probes for geosciences

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Talk Abstract

New ways of measuring the Earth's surface and interior, using probes from naturally occurring radiation, are emmerging in the XXI century, giving rise to Particle Geophysics. The characteristics of the interactions of each elementary particle with matter, govern the information that they can bring, and also our difficulty in accessing it. Muons produced by the interaction of cosmic rays in the atmosphere can travel through large amounts of rock and can be used to image and monitor the structure of vulcanoes or of the subsurface when measured from underground telescopes. Neutrinos created by the same cosmic rays can only be detected in much larger dedicated observatories, but image the full planet. Lower energy neutrinos are created in radioactive decays giving a direct measurement of the global radioactive contents of the Earth. To fully interpret the data, a strong interplay between particle physicists and geoscientists is mandatory, and is increasing. We will review the global efforts in this direction, and give specific example of the ongoing projets in Portugal.

Keywords: Cosmic rays, muon tomography, geoneutrinos, particle geophysics.

Acknowledgements

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Invited speakers

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Dynamics of Self-Organization in Physical and Social Systems

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Talk Abstract

In this talk, we discuss experiments and theory of self-organization in some physical and social dissipative systems. The examples treated here range from fluid flow, pattern selection in fluid–solid systems to chemical-reaction-induced aggregation in fluid systems. We also present some new ongoing work related to self-organization in sports, specifically futbol; which shows parallels with our observations in physico-chemical systems. In each case, self-organization can be seen to be a function of a persistent internal gradient. One goal of this talk is to hint at a common theory to explain such phenomena, which often takes the form of the extremum of some thermodynamic quantity, for instance the rate of entropy production, such as the ones proposed by the Nobel-prize winning work of Lars Onsager and Ilya Prigogine. The arguments have evolved since then to include systems of higher complexity and for nonlinear systems, though a comprehensive theory remains elusive. Our larger attempt, in this research program, is to bring out examples from various disciplines that reveal deep connections between variational principles in physics and biological, or living systems.

Keywords: Self-Organization, Dynamical systems, Networks, Modularity.

Some homogenization results for integral functionals with generalized growth

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Talk Abstract

I will present some recent results related to homogenization problems formulated in terms of integral functionals, both in the context of thin micromagnetic structures and in terms of energies with nonstandard growth conditions.

Keywords: homogenization, generalized growth, Young measures, micromagnetics.

Acknowledgements

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Harvesting populations in random environments: Shortcomings of the optimal harvesting policy, proposal of alternatives, and their assessment

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Talk Abstract

In a randomly varying environment, the dynamics of a harvested (say, a fish) population with size X(t) can be described by a stochastic differential equations – SDE (see, for instance, [1,2]). Let the harvesting profit per unit time (p.u.t.) $\Pi(t)$ be the difference between the selling price p.u.t. of the harvest (assumed proportional to the harvesting yield H(t) =qE(t)X(t), where q is the catchability and E(t) the harvesting effort) and the costs p.u.t. of harvesting (assumed to be a quadratic function of E(t)). Using stochastic control theory, one can determine the optimal harvesting policy $E^*(t)$ $(0 \le t \le T)$, i.e. the policy E(t)that maximizes the expected discounted profit $V = \mathbb{E}\left[\int_0^T e^{-\delta t} \Pi(t) dt\right]$, where $\delta > 0$ is a discount rate. Illustrating with the logistic growth model and parameters provided in [5] for the Pacific halibut, we show that, due to the fast and abrupt random variations in the harvesting effort associated with the environmental induced population size fluctuations, such an optimal policy is incompatible with the logistics of fisheries and therefore inapplicable. It also causes social problems, such as fishermen's unemployment during periods of no or low harvesting. Furthermore, it requires knowledge of the population size at each instant, and estimating population size is an inaccurate, lengthy, and expensive task. To overcome some or all of these shortcomings, we have proposed alternative sub-optimal harvesting policies and assessed them in terms of applicability, possible social problems, and profit (see [2–4]). Among the alternatives, we consider:

(a) constant effort policies, in which $E(t) \equiv E$;

(b) stepwise effort policies, in which E(t) is determined at the beginning of each year (or biennium) according to optimal control theory and kept constant throughout that year (or biennium);

(c) *penalized effort policies*, which incorporate a running energy artificial cost based on deviations of the effort from a reference value in order to avoid abrupt effort changes and periods of low or no fishing;

(d) *stepwise penalized effort policies*, which apply the stepwise procedure to penalized effort policies.

Some of these are nearly optimal and simple to implement in practice. We have also studied the case of populations with Allee effects (see [3]), not reported here due to time constraints.

Keywords: harvesting models, stochastic differential equations, logistic growth, sub-op-timal harvesting policies.

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Attractors and Complexity on Pattern Iterations of Flat-Topped Tent Maps

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Talk Abstract

In [1], it was introduced a method of chaos control designated as "control with simple limiters". The general idea of the procedure is to add an external load to the system, which limits the phase space in such a way that the orbits in the forbidden area are eliminated. Since its proposal, control with simple limiters has been widely used in the control of chaos in areas as diverse as cardiac dynamics, [2], telecommunications or electric converters, [8], population dynamics, [4], or market dynamics [3]. In the one-dimensional case, this procedure leads to the introduction of one (or more) flat segment(s) in the one-dimensional map, with the value(s) of the map on the constant thread(s) corresponding to the limiter(s). Very often, we find modeling situations wherein evolutionary equations have to depend explicitly on time, through time-dependent parameters. This is the case, for example, when we want to model populations with time-dependent forcing or to mimic some control or regulation strategies. In this presentation, based in [5], [6] and [7], we will consider the introduction of simple limiters u in the tent map, in the moments where a binary sequence s (the iteration pattern) is 0. We call to these non-autonomous dynamical systems, pattern iterations of flat-topped tent maps. We will define local and Milnor attractors in this nonautonomous context and study the dependence of their existence and coexistence on the value of the limiter u and on the pattern s. Using symbolic dynamics, we will be able to characterize the families of pairs (u, s) for which these attractors exist and coexist, as well as fully describe them. We will observe that this non-autonomous context provides a richness of behaviors that are not possible in the autonomous case. Finally we will discuss dynamical complexity in pattern iterations.

Keywords: non-autonomous dynamical systems, interval maps, attractors, symbolic dynamics, bifurcation diagrams.

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Data transformations, aggregations and transfer functions for modelling biological and ecological complexities

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Talk Abstract

Biological and ecological systems offer a lot of complexity which should be well understood before we can make valuable regulations. We will address both complex and extreme measurements from given systems. There is a necessity to classify appropriate learning mechanisms and define transfer functions and statistics. I will address learning mechanisms of data transformation and aggregation [3]. In particular, I will introduce SPOCU transfer function [1] and provide some of its unique properties for processing of complex data. Statistical learning will be discussed and tuning of parameters of SPOCU based neural networks will be given. Attractive applications to biological systems e.g. mass balance in the ecosystem of glaciers in Patagonia [2], or methane emissions from wetlands will be addressed.

Keywords: complexity, networks, ecology.

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Numerical simulations using mass-diffusive compressible fluids flows models

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Talk Abstract

This contribution presents first numerical tests of some recently published alternative models for solution of viscous compressible and nearly incompressible models. All models are solved by high resolution compact finite difference scheme with strong stability preserving Runge-Kutta time stepping. Three simple but challenging computational test cases are presented, based on the double-periodic shear layer, Taylor-Green vortex and the Kelvin-Helmholtz instability. The obtained time-dependent flow fields are showing pronounced shear and vorticity layers being resolved by the standard as well as by the new mass-diffusive modified models. The preliminary results show that in some cases the new models are a viable alternative to the well established classical models. Theoretical analysis and numerical solution of various fluids flows problems poses a challenging problem. The widely used mathematical models describing the compressible fluids flows and incompressible fluids flows are the Navier-Stokes-Fourier and the incompressible Navier Stokes systems respectively. These are mixed type systems of non-linear strongly coupled partial differential equations of hyperbolic, parabolic and elliptic type. Their mathematical analysis as well as numerical solution remains one of the most difficult problems of contemporary science. Recently there have been attempts to revise and possibly improve the traditional mathematical models describing the fluids flows. The works of Brennen [2] and Svärd [6] are example of such possible model updates. In these new models, the basic physical principles (conservation/balance laws) are still being used, but the interpretation of certain physical variables and processes brings other options for into the mathematical formulations of such revised models. These changes are bringing some interesting results from the point of view of mathematical analysis [4] of the corresponding models as well as possible increase in the efficiency of numerical methods [3], [5]. The aim of this paper is to present the initial results of a computational study based on the mass-diffusive compressible and nearly-incompressible fluids flows models based on the works of Svärd [6] extended by Kajzer & Pozorski in [5]. The new alternative models are first presented, side by side with the standard systems for both compressible and incompressible fluids flows. The new, mass-diffusive models are then solved by highresolution compact finite-difference methods. The model results are mutually compared for two test cases, documenting the agreement and comparative advantages of the newly formulated models. The full system of *Navier-Stokes-Fourier (NSF)* equations describing the flow of a compressible heat conducting fluid can be written as

$$\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0 \tag{1}$$

$$\partial_t(\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v} + p \mathbb{I}) = \boldsymbol{\nabla} \cdot \mathbb{S}$$
⁽²⁾

$$\partial_t E + \boldsymbol{\nabla} \cdot \left((E+p) \boldsymbol{v} \right) = \boldsymbol{\nabla} \cdot \left(\mathbb{S} \cdot \boldsymbol{v} + \kappa \boldsymbol{\nabla} T \right)$$
(3)

Here the v is the fluid velocity, ρ density, p pressure. The fluid total energy E is defined as

$$E = \frac{1}{2}\rho|\boldsymbol{v}|^2 + \frac{p}{\gamma - 1}$$
 where $\gamma = \frac{c_p}{c_v}$ (4)

for the perfect gas obeying the state equation $p = \rho R T$ with the gas constant $R = c_p - c_v$ being linked to heat capacities c_p , c_v at constant pressure and volume respectively. The stress tensor S for Newtonian fluid is then defined as

$$\mathbb{S} = \mu \left(\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^T - \frac{2}{3} (\nabla \cdot \boldsymbol{v}) \mathbb{I} \right) .$$
(5)

The dynamic viscosity μ and heat conductivity κ depend on the fluid considered. The standard *NSF* system (1)–(3) was reformulated by Svärd [6] who replaced it by a *modified Navier-Stokes-Fourier (M-NSF)* system, having similar form, but different right-hand sides in all equations.

$$\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = \boldsymbol{\nabla} \cdot (\nu \, \boldsymbol{\nabla} \rho) \tag{6}$$

$$\partial_t(\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v} + p \mathbb{I}) = \boldsymbol{\nabla} \cdot (\nu \, \boldsymbol{\nabla}(\rho \boldsymbol{v})) \tag{7}$$

$$\partial_t E + \boldsymbol{\nabla} \cdot \left((E+p)\boldsymbol{v} \right) = \boldsymbol{\nabla} \cdot \left(\nu \, \boldsymbol{\nabla} E \right) \tag{8}$$

The most notable change, probably, is the added mass-diffusive term in the equation for density (6). But the right-hand sides in the momentum and energy equations (7) and (8) have changed as well, consisting now just from the divergence of the gradient of the conserved quantity (in the same form as in the modified mass conservation (6)). The diffusion coefficient $\nu = \mu/\rho$ has now the same value in all the considered equations of the modified *M-NSF* system. The numerical simulations of the alternative compressible as well as nearly incompressible flows models shown their potential in solving problems of practical interest. Although the presented new models offer certain advantages over their classical counterparts (better analytical properties, easier and more efficient numerical implementation), there are numerous issues to be addressed. One of the possible troubles may come from the formulation of the (stress tensor on the) right hand side of the mass-diffusive *M-NSF* model (7). The principle of material frame indifference and the conservation of moment of momentum require the stress tensor to be symmetric (depending just on the symmetric part of velocity gradient). This and many other properties will be in the focus of our future investigation.

Keywords: compressible Navier-Stokes, nearly incompressible flow, mass diffusion, compact finite-difference.

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Contributed talks

Stabilities of Ulam-Hyers type for a Class of Nonlinear Fractional Differential Equations with Integral Boundary Conditions in Banach Spaces

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Talk Abstract

Motivated by the knowledge of the existence of continuous solutions of a certain fractional boundary value problem with integral boundary conditions, we present in here –in a unified manner– new sufficient conditions to conclude the existence and uniqueness of continuously differentiable solutions to this fractional boundary value problem and analyse its stability in the sense of Ulam-Hyers and Ulam-Hyers-Rassias. After presenting the main conclusions, two illustrative examples are provided to verify the effectiveness of the proposed theoretical results.

Keywords: Ulam-Hyers stability, Ulam-Hyers-Rassias stability, fractional boundary value problem, integral boundary conditions.

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On the Green's function for a fractional differential problem and a consequent result

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Talk Abstract

The Fractional calculus is a branch of mathematics that studies the extension of the concepts of derivative and integral to an arbitrary order (necessarily fractional). This definition departs from the traditional concept of derivative and integral in a differential and integral calculus [1]. In this talk, a boundary value of the fractional type with the Riemann-Liouville derivative is presented. By obtaining a Green's function, the solution of the problem is described in terms of an integral equation. Studying its maximum value, we can obtain a Lyapunov-type inequality [2]. This result constitutes a necessary condition for the existence of non-trivial solutions for the boundary value problem under study. Inequalities of this type [3] are proved to have applications in various problems related to the theory of differential equations [4].

Keywords: fractional differential equation, Green's function, Lyapunov inequality, Riemman-Liouville derivative.

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Partially commuting switched systems

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Talk Abstract

This work deals with the stability of switched systems that allow discontinuous jumps (resets) on certain state components during switching instants. If all state components are available in the reset application, the system can be effectively stabilized by selecting the appropriate resets. Stability may not be ensured, though, if some state components are forbidden from resetting. Using a partial state reset, we present a sufficient condition to ensure the stability of a switched system under any switching signal. We formulate this condition in terms of a block-simultaneous triangularizability requirement. According to this, we prove that a partial state reset can stabilize a class of systems with partially commuting stable system matrices. In addition, we propose an algorithm for enabling analysis to determine whether a switched system is one of this specific class.

Keywords: invariant subspace, partially commuting matrices, state reset, stability, switched system

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The dynamical behavior of a piecewise oscillator

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Talk Abstract

Oscillations have been always present either in engineering devices or in physical systems. It is therefore important to understand its nature and sometimes to control them. Since differential equations are the mathematical model most commonly used for both engineering systems and natural phenomena, it is easily understandable why the study of the complexity of its solutions has been a subject of research for the last decades (see, for example, [1] and [5]). In this talk we consider a forced damped piecewise linear oscillator whose motion is modeled by a second-order non-autonomous differential equation. The system has a continuous regime, where the time flow is characterized by the explicit solutions of the ordinary differential equations, and a singular regime, where the time flow is characterized by an appropriate transformation linking the explicit solutions from one domain to the other. So, our system is globally nonlinear and presents complex behavior which is studied using numerical simulations, with similar techniques as the ones applied in [4]. From our previous works, [2] and [3], we know that, for large subsets of the parameter space the oscillator motion can switch rather wildly from a regular to a complex dynamics with a subtle change of parameters. Now we determine regions of the parameter space where the Poincaré map, which describes the observed motion of the forced damped oscillator, can be classified as *m*-modal map, for given *m* natural.

Keywords: dynamical systems, piecewise linear oscillator, interval map, chaos.

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Variationals Inequalities for Obstacle Problems with General Growth

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Talk Abstract

Euler-Lagrange equations play a foundamental role when studing the Sobolev regularity of solutions to variational problems such as

$$\min_{v \in K} \int_{\Omega} F(x, \nabla v) dx,$$

with $\Omega \subset \mathbb{R}^n$ open and bounded, and K subset of some appropriate Sobolev space. It is well known that when the Lagrangian function satisfies p-growth conditions or (p,q)growth conditions then it is possible to prove that the solutions satisfy a related Euler-Lagrange equation. In the case of the obstacle problem, due to the constraint on the solutions, it is only possible to write a related variational inequality. In this talk I show how to prove the aforementioned variational inequality when working with more general growths. This study was inspired by [1] and [2]. For our work [3] we assume only superlinear growth for the Lagrangian function F.

Keywords: Variational inequalities, minimizers, obstacle problems, convex extension, monotonic approximation.

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Analysis of The Art of Fugue: capturing Bach's style with mathematical patterns and transformations

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Talk Abstract

The Art of Fugue is a collection of 20 fugues and cannons written by Johann Sebastian Bach in the 1700s. Each piece starts with a single theme which is transformed, developed and mixed in fascinating mathematical combinations resulting in a distinctive musical style. In this talk, we use the method introduced in [1] to analyze 20 pieces from the Art of the Fugue. We focus on the variation of motifs from one piece to another as well as the complexities of mathematical transformations present throughout the entire collection. We also demonstrate how using maps of transformations could guide non-musicians to compose original pieces in the style of Bach's fugues.

Keywords: Bach, fugue, motif, geometrical transformations, math and music.

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Factorisation of the classical nonstandard bounded functional interpretation

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Talk Abstract

Functional interpretations are maps of formulas from the language of one theory into the language of another theory, in such a way that provability is preserved. Functional interpretations have many uses, such as: relative consistency results, conservation results and the extraction of computational content from proofs. We prove the factorisation U = KB of Jaime Gaspar and Fernando Ferreira's classical nonstandard bounded functional interpretation U [2] in terms of Jean-Louis Krivine's negative translation K [5] and Bruno Dinis and Jaime Gaspar's intuitionistic nonstandard bounded functional interpretation B [1]. We also give some applications of the factorisation.

Keywords: factorisation, bounded functional interpretation, negative translation, nonstandard arithmetic.

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On the properties of measures of relative variability

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Talk Abstract

When comparing the variability of several datasets, whose values are expressed in different measurement units, is convenient to use a measure of relative variability. One common measure of relative variability is the coefficient of variation, which expresses the standard deviation as a proportion of the mean and does not depend on the unit scales. This measure can be applied for various purposes (as a measure of risk sensitivity, to represent the reliability of trials, and others), having important applications in research in agriculture, industry, medical and social sciences, education, and many other fields (e.g. Cox and Sadiraj[1], Reed et al.[2], Romano et al.[3], Weber et al.[4]). The assumptions for using the coefficient of variation are mostly related to the nature of the data, however some of the properties of this coefficient lead to limitations that affect its suitability in certain situations. In this work we address the properties of the coefficient of variation, analysing an alternative to the common coefficient of variation, introduced by Dodd[5], which allows us to overcome its disadvantage of dependence on sample size.

Keywords: bounds, corrected coefficient of variation, dispertion.

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Algebraic structure for recombining cellular automata

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Talk Abstract

We develop an algebraic structure arising from recombination of cellular automata. Cellular automata are discrete dynamical systems with local state \mathbb{Z}_n and a particular cellular automaton is characterized by a finite sequence of elements in \mathbb{Z}_n . This sequence determines the time evolution rule and it is seen as the genotype of the cellular automata. We have defined in [1] a binary operation which determines the recombination of two finite sequences, each associated to a cellular automata. This operation is parametrized by real number, α , in the unit interval. Therefore, we obtain a one parameter family of algebraic structures defined on the space of cellular automata. This algebraic structure is non-commutative and non-associative. The main objective of our work is to study the algebraic structure generated by a finite initial population of cellular automata through recombination. Moreover, we study the Cayley graph for the structure, which can be seen as the phylogenetic tree for the initial population. We discuss the structure dependence of the parameter α , and the maximum diversity that can potentially be obtained, given a finite set of generators, and how fast it can be attained.

Keywords: cellular automata, recombination, algebraic structure, maximal diversity.

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Rank and related tests for grouping factor levels: An application to cocoa breeding experiments

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Talk Abstract

In the early 1930s, R. A. Fisher introduced an exact method for hypothesis testing by permuting data without changing its distribution under the null hypothesis. These permutations tests form the basis of classical nonparametric statistics, which itself is commonly associated with rank tests. Due their robustness against assumptions about the underlying data distribution, rank tests have gained popularity in many research fields, including agriculture and finance, to name just a few. In 2023, Opoku-Ameyaw et al. proposed a nonparametric test for grouping levels of a factor. This test was applied to a cocoa breeding experiment in Ghana, to evaluate the performance of adaptability of twelve varieties of cocoa on four types of acidic soils. There were three groups of varieties, two of which were of a common ascendant, and the homogeneity of these two groups was tested. The results revealed that the grouping is significant for the most acidic soil. The present study proposes a randomization procedure inspired by Fisher's randomization method, which aims to derive the exact distribution of the test statistic considered in Opoku-Ameyaw et al. (2023), focusing on both univariate and multivariate cases. The obtained results corroborate the findings of the previous work, providing further evidence of the applicability and relevance of the proposed approach.

Keywords: Nonparametric test, Fisher's randomization method, univariate test, multivariate test, cocoa breeding experiment.

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Optimisation and Analysis of the Perovskite Solar Cell

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Talk Abstract

The search for better conversion efficiency and the low production cost of solar cells has sparked the interest of several researchers with the aim to find new materials to meet the global challenges of the energy matrix [1–3]. This has made the inorganic halide, Cesium Lead lodide ($CsPbI_3$), one of the most promising due to its stability and, on the other hand, better efficiency compared to other perovskites [4,5]. In this study, a perovskite solar cell was optimised with a configuration featuring $CsPbI_3$ as the active layer, TiO_2 responsible for electron transport (ETL), Spiro-OMeTAD responsible for the gaps (HTL). The SCAPs-1D software was used for the simulations where the physical parameters were introduced, these parameters were deduced from cross-referencing various studies of experimental and published literature. As a result of this investigation, a significant increase in the cell's performance was obtained. The configuration of the perovskite cell structure can be well optimised to increase conversion efficiency and make them more preferred in the market.

Keywords: Optimization, *CsPbI*₃ perovskite, SCAPS, PCE, Solar cells

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Exploring Variability in Individual Growth: Introducing a Hierarchy of Stochastic Models

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Talk Abstract

In a scenario involving M individual animals experiencing random environmental fluctuations, a broad class of individual growth models, initially introduced by O. García [1], relies on the stochastic differential equation (SDE)

$$dY_i(t) = \beta(\alpha - Y_i(t))dt + \sigma dW_i(t), \ (i = 1, \dots, M),$$
(9)

where $X_i(t)$ represents the size of the *i*-th individual (i = 1, ..., M) at age t, $Y_i = h(X_i)$ is a transformed size via a strictly monotonous C^1 function h, $\alpha = h(A)$ denotes the transformed value of the asymptotic size A, $\beta > 0$ is a growth parameter, $W_i(t)(i = 1, ..., M)$ are independent standard Wiener processes, and $\sigma \ge 0$ quantifies the strength of environmental fluctuations on growth. In previous work, we have extensively investigated this model for parameter estimation, profit optimization in livestock production, and other related topics. However, there is the possibility of individual variability, wherein different individuals possess distinct parameter values, denoted as α_i and/or β_i (i = 1, ..., M). This leads to a mixed SDE model, characterized by

$$dY_{i}(t) = \beta_{i}(\alpha_{i} - Y_{i}(t))dt + \sigma dW_{i}(t), (i = 1, \dots, M),$$
(10)

with $\alpha_i = \mu + \sigma^* \epsilon_i$, where ϵ_i are i.i.d. standard Gaussian random variables (and/or similarly for β_i). Parameter estimation for mixed models often requires approximate methods, as demonstrated in [2,3], where we have developed the delta approximation method and compared it with existing techniques. Moreover, individual parameter values, such as α_i , may vary according to specific characteristics (e.g., genetic values) of the individual. Here, we will illustrate with the linear dependence on one genetic value g_i , so that $\alpha_i =$ $\mu + cg_i + \sigma^* \epsilon_i$. This leads to the formulation of more intricate models and provide more individualized predictions. By employing a hierarchy of models with increasing complexity, we can systematically evaluate the significance of various sources of variability. Here, we will focus on the case of α varying randomly among animals. Testing a hierarchy of hypotheses related to environmental variability effects ($\sigma = 0$), parameter variability among individuals ($\sigma^* = 0$), and the impact of a specific characteristic (e.g., genetic value) (c = 0) facilitates a comprehensive understanding of the underlying dynamics. We will illustrate statistical methods for detecting significant sources of variability and estimating parameters using real data from a large dataset of bovines from the Mertolengo breed.

Keywords: genetic traits, individual growth, mixed models, stochastic differential equations.

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Convertibility of singular symmetric matrices

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Talk Abstract

In this work, a relation between the determinant and the permanent on singular symmetric matrices is established. Several no go theorems have been proved, and some of them related with the conversion of the determinant into the permanent for symmetric matrices and singular matrices separately. The aim of this paper is to reinforce the lack of solution to the convertibility problem concerning to singular symmetric matrices, more concretely, being $H_n(\mathbb{F})$ the linear space of *n*-square symmetric matrices over a field \mathbb{F} with at least *n* elements and whose characteristic is not 2, we show that if $n \geq 3$ there is no linear transformation

$$T: H_n(\mathbb{F}) \to H_n(\mathbb{F})$$

satisfying the condition det(X) = 0 if and only if per(T(X)) = 0, for all $X \in H_n(\mathbb{F})$.

Keywords: linear preserver problems, permanent, determinant, symmetric matrices.

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A comprehensive model for computation and measurement numbers

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Talk Abstract

Historically, a distinction has been made between numbers serving for computations, which are intrinsically mathematic, and numbers serving for measurements [3] of, say, physical quantities. Attempts for modelling have been made by among others Laugwitz [4] and Spalt [6]. Here we aim at an interpretation of computation numbers and measurement numbers using an elementary extension of Nelson's axiomatics as presented in [5]. This nonstandard axiomatics assigns the predicat "standard" only to natural numbers, and concerns a weak fragment of Nonstandard Analysis; Nelson argues that his axiomatics is sufficient to develop advanced stochastics avoiding measure theory, in some exceptional cases he uses a "star" axiom, which corresponds to sequential saturation. The axiomatics permits to define standard integers, rational numbers and algebraic numbers, which could serve as a model for numbers for concrete computations. However it is not possible to incorporate standard real numbers in this approach. We show that the "star" axiom and an elementary fragment of Kanovei and Reeken's axiomatics for external sets HST enable us to define standard real numbers as equivalence classes. Its elements differ at most infinitesimally, and this imprecision makes them a model for measurement numbers. The approach bears some relation with the work on external numbers in [2] and flexible meadows in [1].

Keywords: Computational numbers, measurement numbers, infinitesimals.

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How Earth's internal structure is characterized by seismic and muon tomographies

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Talk Abstract

In the study of Earth physics, mathematical theories are employed to quantify and model the physical properties of the inner Earth. In this work, two different methodologies, widely used in geophysics, are presented: seismic and muon tomography. The seismic tomography determines the velocity of seismic wave propagation, while the muography – as a new geophysical tool – measures the transmission of muons, which are consequently converted into densities. These two techniques complement each other and provide information about different rock properties (densities and velocities), related by empirical formulas. In seismic tomography, we use earthquakes or vibrations emitted by humans to record the seismic waves' arrival times at the seismometers implemented around the area of interest. We talk respectively about passive and active seismic tomography. In the first case, two inverse problems need to be resolved: 1- the earthquake localization, determining the position and origin time of the earthquake, and 2- the computation of the velocity of seismic wave propagation. On the other hand, active seismic tomography consists of installing geophones and creating vibrations in different ways to create a distribution of seismic rays in an area intended to be explored. In that case, one unique problem needs to be resolved: predicting the velocity distribution in two or three dimensions. Both problems have the objective of understanding the structure of the Earth's interior, having a resolution of some kilometers, for passive inversion, and a few meters, for active seismic tomography, using the same inverse theory with different degrees of control on

the model resolution. The second approach, the muography, uses the natural emission of muons coming from the interactions between cosmic rays and the atmosphere to study the structure of mines, volcanoes, and archeological sites. The muographs describe the distribution of the average density deduced from the attenuation of the muon flux, measured using a muon telescope while crossing a media. The recording of the accumulated muon detections forms a muon direction map reflecting the muon flux's attenuation while crossing a specific material. This attenuation is proportional to the density of the crossed material and/or the amount of detected muons. Therefore, areas traversed by a large number of muons will be related to low densities, while those with high density will detect a poorer quantity of muons. To obtain high-resolution muographs (i.e. "muon r-ray" images), the detection maps are normalized using the open sky muon flux. Merging this information and the distance that the muon will travel through a media, muon counts can be translated to average rock densities calculated within the column of matter being crossed in each direction. Both models are obtained by employing the inverse theory, which is non-linear for seismic inversion and linear for muography, using different mathematical approaches. The inverse problem is generally applied to predict a model based on a specific dataset. In this approach, various models can be obtained from the same set of data leading to the non-uniqueness of the solution. This means that, due to the errors present in the data, some model details cannot be resolved. This barrier in the resolution of the inverse problem is accepted, and mathematical and physical theories as well as prior information about the model, should be used to constrain the solutions and give more realistic inverted models. As an outcome, these two geophysical approaches are joined together to converge to a complete and reliable interpretation of the Earth's structure and lower its uncertainty.

Keywords: inverse problem, active seismic tomography, passive seismic tomography, muon tomography.

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Meadows - On how to divide by zero

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Talk Abstract

Meadows are algebraic structures, similar to rings, but where division by zero is allowed. These structures were introduced by J. Bergstra and his co-authors in a series of papers in the context of Computer Science. The two main classes of meadows are involutive meadows, where the inverse of zero is zero, and common meadows, where the inverse of zero is an element that is absorbent with respect to both the sum and the product. In this talk we give a brief introduction to common meadows and discuss some of their algebraic properties. In particular, we study the problem of enumerating finite common meadows, which is related with concepts that arise in the study of combinatorial structures and in number theory. Some open questions on this topic will also be discussed.

Keywords: Common meadows, unital commutative rings, enumeration of finite structures.

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Exploring Linear Elasticity: Unveiling the Power of Physics-Informed Neural Networks (PINNs)

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Talk Abstract

Linear elasticity, a fundamental theory in solid mechanics, describes the deformation behavior of elastic materials under external forces. The governing equations, typically comprising partial differential equations (PDEs) such as the equilibrium equations, stressstrain relations, and boundary conditions, form the backbone of predicting stress and displacement fields within elastic bodies. Solving these equations analytically is often infeasible for complex geometries and loading conditions, necessitating the use of numerical methods. Traditional numerical methods like the Finite Element Method (FEM) have been extensively employed to tackle linear elasticity problems. However, these methods can be computationally intensive, particularly for high-dimensional and largescale problems, and often require fine meshing to achieve accurate solutions. Moreover, they can struggle with incorporating data-driven insights seamlessly, which is increasingly important in modern engineering applications that leverage real-time data for predictive maintenance and optimization. Physics-Informed Neural Networks (PINNs) have emerged as a promising alternative, offering a novel approach that integrates the power of deep learning with the rigor of physical laws. PINNs utilize neural networks to approximate the solution of PDEs, embedding the governing equations and boundary conditions into the loss function during training. This fusion of data-driven modeling and physics-based constraints enables PINNs to provide solutions that respect the underlying physical principles, potentially offering several advantages:

Mesh-free Solutions: Unlike FEM, PINNs do not require a mesh, which simplifies preprocessing and can handle complex geometries more flexibly;

Data Integration: PINNs can seamlessly integrate experimental data and simulation results, enhancing predictive accuracy and enabling real-time updates;

Scalability: The inherent parallelism of neural networks can lead to more scalable solutions for large-scale problems;

Adaptability: PINNs are easily adaptable to different types of boundary conditions and material behaviors without the need for significant reprogramming.

In the context of linear elasticity, PINNs can be particularly advantageous for solving problems where traditional methods face challenges. For example, in 2D elasticity problems involving irregular domains, varying material properties, or complex loading conditions, PINNs can provide robust and efficient solutions. With this talk, we present several applications of PINNs in 2D (and 3D) linear elasticity to demonstrate their effectiveness and versatility.

Keywords: Linear elasticity, Physics-Informed Neural Networks, Data-driven scientific computing, Machine learning, Predictive modeling.

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requisitions

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Talk Abstract

The DigitArg platform is the Portuguese National archive system that uses well-established description standards, namely the ISAD(G) (General International Standard Archival Description) and ISAAR(CPF) (International Standard Archival Authority Record for Corporate Bodies, Persons and Families) with a hierarchical structure adapted to the nature of archival assets. In the EPISA project, one of the tasks included the migration of the DigitArg information into a linked open data model, CIDOC-CRM [5]. This task included the representation of textual description in the ISAD(G) element 'Scope and Content' by extracting the information from natural language written text. The dataset for handwritten recognition has 1000 registers with: digital representation, a text description of the digital content, and the semantic representation in CIDOC-CRM of the text description [6]. This information enables the automatic evaluation of handwritten recognition and can be used to improve the performance of handwritten recognition through the use of semantic information. The handwritten data was selected from a set of registers with digital representation, a jpg file, from the Portuguese National Archive. The registers were chosen from those that have a text transcription of digital representation in the DigitArg platform. Handwritten text recognition is an important task in computer vision that has received considerable attention in recent years [1,2]. In our approach, the open-source document processing platform ArkIndex [3,4] (https://teklia.com/our-solutions/arkindex/) is used to automatize the document recognition system adapted to the passport registers with digital representation. Initially, a corpus of 100 registers was built up and a manual annotation was performed to represent the structure of the pages (text zones, pages and text zones transcriptions), producing an automatic transcription of the handwritten text. The described approach evaluation reveals promising results that confirm that the initial annotated corpus can be used to obtain a general tool for processing the passport registers in DIGITARQ.

Keywords: handwritten recognition, document annotation, artificial intelligence, data analysis.

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Accounting for Bias in Extreme Value Index Estimation: Applications in Environmental Science

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Talk Abstract

The field of Extreme Value Statistics primarily focuses on estimating parameters related to extreme events, such as the probability of surpassing a high threshold or an extreme quantile. The estimation of the extreme value index (EVI) holds significant importance within this field, offering crucial insights into distribution tail behavior. The Hill estimators [1] are commonly utilized methods for EVI estimation, particularly in models featuring an EVI > 0 (heavy tails). However, the susceptibility of these estimators to bias poses a challenge, potentially leading to inaccuracies in EVI estimations. Such inaccuracies can significantly impact the reliability of risk assessments and decision-making processes. To address this issue, this paper proposes a new reduced-bias EVI-estimator, in the lines of [2], based on a particular generalized mean [3] and in the recent work [4]. The methodology will be applied to several sets of real environmental data and to simulated data for validation.

Keywords: statistics of extremes, reduced bias estimators, extreme value index, environmental data.

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A Numerical Approach to Pfaffian Equations

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Talk Abstract

Although being a part of a well-established classic area in Differential Geometry and Algebra, Pfaffian differential equations are somewhat unknown these days, at least in Analysis. Although formally formulated by Pfaff [3], the problem was already known to Euler [2]. From an analytical viewpoint, it has been traditionally treated in textbooks dealing with Differential Equations focusing on analytical techniques to understand and, eventually, find solutions. The success of those methods are however limited to explicit situations where computations can be carried out. There are some general existence and uniqueness results as well. Yet the numerical approximation of such solutions has not been, as far as we know, treated. By means of a variational approach based on a true vector variational problem, we propose a mechanism to examine and numerically approximate such solutions [1].

Keywords: differential equations, vector variational problems, conjugate gradient method.

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Statistical modeling of burnout in college students as a second order construct

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Talk Abstract

Burnout in college students has been studied considering a three-factor structure, in line with the conceptualization proposed by the Maslach Burnout Inventory - Student Survey (MBI-SS) scale. The expansion to a second-order hierarchical structure has also been proposed with Confirmatory Factor Analysis and the maximum likelihood method. Here we considered a higher order structural equation model (SEM), where burnout in students is a 2nd order construct, and used the consistent Partial Least Squares (PLSc) estimator, which performs a correction of reflective constructs' correlations to make results consistent with a factor-model. At a Portuguese Polytechnic Institute, an online survey was carried out using the 15 questions of the MBI-SS, in which the data from the sample collected (n = 144) did not present a multivariate normal distribution. Thus, we applied the PLSc-SEM, which is a non-parametric method that makes no distributional assumptions and can be used with relatively small sample sizes. In particular, we applied the disjoint two-stage approach to estimate a reflective-reflective second order model of students' burnout with "emotional exhaustion", "disbelief" and "personal effectiveness" as first-order constructs. The internal consistency values for the first-order dimensions are quite reasonable (higher than or equal to 0.799). The estimated model, which verified the key criteria in the evaluation process of the outer and inner submodels, can then be used to obtain the global burnout score of students at the institution under analysis.

Keywords: higher-order constructs, likert-type scale, partial least squares, structural equation model, survey.

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Optimizing MPPT: A Comparative Study of P&O, Predictive Control, Fuzzy Logic, and Neural Network Methods

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Talk Abstract

Optimizing the energy efficiency of solar PV panels is pivotal for advanced energy harvesting and utilization. Solar PV panels energy generation depends on environmental parameters such as irradiance and temperature which can increase the maximum power point tracking (MPPT) algorithm complexity due to its nonlinear characteristics. This study investigates a comparative analysis of four MPPT algorithms based on P&O, Predictive Control Method, Fuzzy Logic, and Artificial Neural Network (ANN) [1]. A robust boost converter is designed along with a classical P&O-based MPPT algorithm and compares the simulation results with artificial intelligence-based MPPT algorithms. The P&O technique is evaluated with the predictive control method which leverages next-state predictions towards enhancing accuracy. The study indicates Fuzzy Logic can manage system uncertainties along with adaptive decisions, while Artificial Neural Networks (ANN) explore the potential to learn dynamically and adapt to changing conditions [2–4]. The simulation shows the variation in response time, efficiency, and stability across the algorithms. This comparative study provides a guide to selecting the most effective and efficient MPPT algorithms to elevate the performance and reliability of solar PV systems [5,6].

Keywords: MPPT, P&O, Predictive Control, Fuzzy Logic, Artificial Neural Network, Photovoltaic Systems, Renewable Energy.

Acknowledgements

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Development of IoT based monitoring and fault detection technique of Hybrid PVT System

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Talk Abstract

The evolved monitoring and fault detection technic of hybrid PVT systems is playing a crucial role in the modern energy management system. The number of PV plants are exponentially increasing and to ensure optimal performance, require continuous observation and maintenance. Unpredictive weather characteristics results uncertainty of the output monitoring system and the non-linearity of the solar cells characteristics result in complicated faults detection using conventional methods [1]. The main purpose of this study is to monitor the PVT system output and identify the faults in real time using IoT. In order to solve these problems, integrating Artificial Intelligence (AI) has become increasingly a common choice [2,3]. The existing monitoring and fault detection systems are expensive, complex and requires high energy investment. An automatic low-cost virtual monitoring data acquisition system with fault detection of hybrid PVT will be developed in this work. The proposed system is able to store, monitor, and display both collected data of the environmental variables including other relevant electrical output parameters. Additionally, it will detect the faults in the panel by analyzing the obtained current-voltage (I-V) and power-voltage (P-V) curve with stored data. It is found relevant to do such experimentation as the monitoring and fault detection helps to improve the PVT system's performance. Additionally, the proposed system will be put in the grid simulator to evaluate its robustness and effectiveness.

Keywords: PVT System, monitoring, fault detection, IoT, MPPT.

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Next-generation Diagnostic Precision: A Tailored Deep Neural Network for Alzheimer's Disease Classification in MRI

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Talk Abstract

Alzheimer's Disease (AD) is a devastating neurological disorder that severely impacts millions of individuals worldwide, leading to significant deterioration in memory, cognitive functions, and behavioral patterns [1]. Early and accurate diagnosis is critical for managing and treating AD effectively [2]. This study introduces a novel AD classification model that combines cutting-edge deep learning techniques with advanced feature extraction [3], adaptive weighting, and optimized hidden layers to achieve unprecedented levels of accuracy and efficiency [4]. Our model leverages deep Convolutional Neural Networks (CNN) for robust feature extraction, focusing on critical biomarkers associated with AD. This approach ensures that the model can accurately capture the subtle differences in brain MRI scans indicative of various stages of AD. By integrating adaptive weights and optimized hidden layers, the model enhances classification performance while maintaining computational efficiency. To address the significant class imbalance in the Kaggle MRI image dataset [5], we employ a synthetic oversampling technique to ensure an even distribution of images across different classes, improving the model's generalizability. The proposed Alzheimer's Disease Detection Network (ADD-Net) is rigorously evaluated against established models such as DenseNet169, VGG19, and InceptionResNet V2 using metrics like precision, recall, F1-score, Area Under the Curve (AUC), and loss. The model achieves exceptional performance metrics: 99.51% accuracy, 99.51% precision, 99.51% sensitivity (recall), 1.000% AUC, 99.51% F1 score, 0.0138% loss, and 99.32% ROC. This research not only advances the field of AD classification but also sets a new benchmark for diagnostic tools. The integration of adaptive weights and optimized hidden layers within a deep CNN framework leads to superior performance across all evaluation metrics. The streamlined architecture of our model demonstrates that high precision and accuracy can be achieved without excessively increasing model complexity or computational demands. The exceptional performance metrics underscore the model's potential as a reliable and powerful tool for AD diagnosis, highlighting its superiority over existing models and setting a new standard in the domain.

Keywords: Deep Learning, Supervised Learning, Image Classification, Imbalanced Dataset.

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Five years of eddy covariance flux measurements in Alqueva reservoir

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Talk Abstract

From 2017 to 2022, micrometeorologic measurements of the three components of the wind together with water vapour and carbon dioxide densities were performed continuoulsy in a floating platform over the Alqueva reservoir. The eddy covariance method was applied to the 20 Hz data to generate 30 minutes fluxes enclosing smaller and bigger turbulent eddies from the atmosphere over the water surface. The resultant fluxes are the momentum, latent heat, sensible heat and carbon dioxide. Results and discussion from the resultant fluxes are presented from seasonal to annual scale and relations with the thermal stratification of the reservoir.

Keywords: Eddy Covariance, Fluxes, Reservoir, Micrometeorology.

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Forecasting solar power output through LSTM-Based Models

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Talk Abstract

The escalating demand for renewable energy solutions especially solar power necessitates precise prediction models to optimize energy production and grid management. Photovoltaic (PV) power generation while sustainable is highly dependent on weather conditions [1]. Making accurate forecasting is a crucial aspect of energy planning and management. This study leverages an extensive dataset from the Verney weather station at Evora, Portugal. The dataset includes key meteorological variables such as solar radiation, temperature, and humidity, crucial for modeling PV output . Our objective was to develop a robust predictive model using Long Short-Term Memory (LSTM) networks to forecast PV power output based on the collected weather data. LSTMs are a form of recurrent neural networks which are particularly suited for time-series data due to their ability to retain information over extended periods and their effectiveness in capturing temporal dependencies within the data. This aspect is crucial for understanding and predicting solar power generation. The methodology involved preprocessing the dataset to handle anomalies and normalize the input features [2]. The LSTM model was trained on sequences of hourly weather data to predict daily PV output. We divided the dataset into training, validation, and testing segments to ensure a comprehensive evaluation of the model?s predictive power. Results from the LSTM model demonstrated a high degree of accuracy in PV power predictions with a root mean squared error (RMSE) significantly lower than traditional time-series forecasting models. The LSTM effectively captured the nonlinear relationships between weather variables and PV output. The obtained results ensure that advanced deep-learning techniques can substantially enhance solar power forecasting [3,4]. The LSTM-based model represents a significant advancement in the predictive analytics of solar energy systems. By accurately forecasting solar power output, utility operators can better manage the variability and intermittency of solar power. This leads to more stable and efficient energy systems. This research shows the potential of deep learning techniques in revolutionizing energy forecasting and contributes to the broader goal of enhancing the reliability and efficiency of renewable energy sources. Future work will focus on integrating more temporal data and exploring hybrid models that combine LSTM with other predictive techniques to further enhance forecasting accuracy [5].

Keywords: RNN, LSTM, PV, Energy.

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New Dynamic Coupling Strategy for Stabilization of Unmanned Swarm Systems

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Talk Abstract

Unmanned Aerial Vehicles (UAVs) or drones have gained significant attention due to their wide range of applications, including surveillance, search and rescue or military operations [1]. More recently the indoor use of drones is considered for wide other reasons such as monitoring of industrial installations or visual inspections. In such environment the drone's navigation suffers from a lack of standard navigation methods based on satellites positioning and even local systems based on Inertial Navigation can be affected by different kind of noises and inherent drift errors. In those conditions a distributed automatic flight system is more accurate and precise considering a single UAV or even a flying swarm formation [2]. Such a system can ensure flight robustness and security for each drone as well as for the environment. In this work a mathematical models based on couplings to suppress chaos and produce a swarm flight stabilization strategy are proposed [3]. A numerical simulation in MATLAB is performed and an experimental setup based on multiple UAV quadcopters is proposed for comparison and physical validation purposes.

Keywords: Unmanned Aerial Vehicles (UAVs), Drones, Surveillance, Unmanned Swarm System.

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Cellular Automata for Analyzing Patterns in Idealized Material Blocks

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Talk Abstract

Cellular automata are computational models consisting of a lattice of cells, where each cell can occupy one state from a finite set. The state of each cell evolves in discrete time steps according to a specific set of transition rules that depend on the states of its neighbouring cells. These models are widely utilised for simulating and analysing dynamic systems and processes across diverse scientific domains [1], [2], [3] and [4]. This study aims to analyse patterns resembling fractures by characterising specific forms within idealised material blocks. We explore a generic probabilistic cellular automaton and apply refinement techniques to optimise probability distributions. This refinement process can integrate empirical data or predefined behaviours, enabling adjustable modifications to the cellular automaton's rules [5].

Keywords: elementary cellular automaton, dynamical systems, probability distribution, patterns.

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Stabilization and Control of an Inverted Pendulum System Using a Microcontroller and Voice Control

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Talk Abstract

Balancing an inverted pendulum is a challenging task and a classic problem in control theory [3,2,1]. Such a system is characterized by its unstability and nonlinearity that requires precise instrumentation and control algorithms [4]. In this work, we present the design and implementation of a stabilized inverted pendulum system using an ESP32 embedded system, a MPU6050 gyroscope, and a PID controller. The MPU6050 gyroscope is used to measure the tilt angle of the pendulum, providing feedback to the control system. The PID gains (Kp, Ki, and Kd) are tuned using a trial and error approach, taking into account factors such as rise time, settling time, overshoot, and steady-state error. The ESP32 embedded system is chosen as the main controller due to its powerful processing capabilities, built-in Wi-Fi and Bluetooth connectivity, and support for various peripherals. The performance of the stabilized inverted pendulum system is evaluated based on its ability to maintain the pendulum's upright position under various conditions, such as external disturbances and changes in the pendulum's parameters. The integration of voice control adds a unique and user-friendly aspect to the system, allowing for intuitive interaction and control of the inverted pendulum.

Keywords: Self-Balancing Robot, Control Theory, Microcontrollers, PID Controller.

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Instrumentation and Signal Processing: Leveraging Metaheuristic Algorithms for Enhanced Performance

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Talk Abstract

Heuristic and metaheuristic optimization algorithms faced a quite significant development in the last decade and can be found in use in several science and engineering applications, like in medical science, robotics, aerospace engineering and image processing [1]. Other particular area where this sort of algorithms, along with other in the wide field of artificial intelligence, find application, is on the solar energy conversion systems [2,3]. These are widely used because of their simplicity to implement, their converge to the global optima and their facility to be applied to various complex optimization problems where the classical methods are not efficient [4]. In instrumentation, frequently it is performed the adaptation of sine signals, from one ore two measurement channels, relying in sine fitting methods; those methods consist in the search of the sinus parameters that adapt better, under the minimum squares criteria, considering a set of data acquired with an analog-to-digital converter (ADC) [5]. When dealing with one input channel, frequently that sine wave data fitting operations are performed via the algorithm for three-parameter (known frequency) least squares, or the algorithm for four-parameter (general use) least squares [6,5]. Regarding those methods, the former is non-iterative and assumes the need of a known frequency, whereas the later is iterative and also estimates the frequency. In what pertains to the three parameters method, the authors have developed previous studies and among them, appears the known frequency accuracy sensitivity assessment; or, how the considered frequency correctness level influences the error obtained in the fitting operations [5]. It was carried out the study of the normalized quadratic error resulting from the method application, depending on the input signal frequency, to several ADC acquired number of samples and varying other conditions; it results in real variable real functions with one global minimum, for whose the number of samples increase motivates the appearance of an increasing number of local minimums. So, the optimization operations on these functions face a challenge with increasing complexity as the number of samples are augmented. It were applied several heuristic and metaheuristic optimization algorithms, in order to optimize those functions, varying not only the ADC number of samples, but also other conditions. There are presented case studies with several scenarios, underlining the results.

Keywords: instrumentation and measurement, sine fitting algorithm, heuristic or metaheuristic optimization algorithm.

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Optical characterization of the São Domingos Mine heaps with satellite imaging

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Talk Abstract

The INCOME Project proposes the creation of an environmental management model for mining areas contaminated by toxic metals. This model integrates the results of the application of a set of analytical methods and instruments for monitoring contaminated areas, such as biophysical analyses, soil, and water chemistry, geophysics, and satellite hyperspectral remote sensing. Data from all research will be used as inputs to be used by artificial intelligence, which will allow the production of contamination models using less data than standard methodologies. The optical characterization of the mining heap using satellite data available from the Copernicus program (EU Space Programme) allows extracting information about the surface to make the correspondence with heavy metals. This work will include the optical analysis of satellite data relating to the surface of the São Domingos Mine (Mértola, Portugal).

Keywords: Optical characterization, Satellite imaging, Remote Sensing.

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Existence and localization of periodic solutions in impulsive systems

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Talk Abstract

We present some existence and localization results for periodic solutions of first and second order nonlinear coupled systems of two equations, with and without impulses. The existence arguments for first-order problems are based on Schauder's Fixed Point Theorem [1] together with the upper and lower solution method. For second order non-impulsive systems, the existence of solutions is assured by a variation of the Nagumo condition and the Topological Degree Theory. For second order impulsive systems, we prove the existence using Green functions and Schauder's Fixed Point Theorem. Two novelties is that periodicity is not required for the nonlinearities, and that the upper and lower solutions need not to be necessarily well-ordered. For the impulsive analysis, results on equi-regulated functions [2,3] are required. We present different applications to illustrate the main results.

Keywords: Impulsive nonlinear systems, upper and lower solutions, existence and localization, periodic solutions.

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Enhancing Ornamental Rocks industry Efficiency: Application of Electric Tomography and Poisson coefficient Modulation for Improving marble Characterization

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Talk Abstract

Electric tomography for ornamental rocks modulation [1,2] specially in marble industry, encounters intricate challenges, notably concerning the determination of conductivity distributions within objects. This work elucidates the nuanced complexities of utilizing the Poisson coefficient within this context, scrutinizing its parameters and components and their implications for marble block modulation. The study holds significant importance in revolutionizing marble characterization and modulation, crucial for various industries such as construction, sculpture, and interior design. By addressing the challenges in determining conductivity distributions within marble blocks, our research aims to pave the way for enhanced efficiency and innovation in the marble sector. The Poisson coefficient [3], a fundamental parameter in materials science and mechanics, plays a crucial role in characterizing the behavior of materials under deformation. In the context of marble block modulation, the Poisson coefficient v describes the ratio of transverse strain to axial strain when a material is subjected to uniaxial stress. The relationship between this coefficient and the conductivity distribution $\sigma(x, y, z)$ within marble blocks can be mathematically expressed as:

$$\nabla\cdot(\sigma\nabla\vartheta)=0$$

where

- $-\nabla \cdot (\sigma \nabla \vartheta) = 0$, represents the Laplacian operator applied to the conductivity σ and the electric potencial ϑ ;
- $-\sigma(x, y, z)$ denotes the conductivity distribution within the marble block, which is influenced by factors such as mineral composition and structural imperfections;
- -v is the Poisson coefficient, which describes the material's response to deformation and its influence on the conductivity distribution;
- $-\nabla \vartheta$ represents the gradient of the electric potential, describing the spatial variation of electric potential within the object.

However, its application to marble blocks is fraught with obstacles due to the material's unique properties and irregular geometry. Marble blocks exhibit heterogeneous conductivity distributions influenced by factors like mineral composition and structural imperfections. Consequently, applying the Poisson coefficient without accounting for these variations can yield inaccurate conductivity reconstructions. Furthermore, the irregular geometry of marble blocks poses challenges. The Poisson coefficient relies on assumptions about boundary conditions and geometric properties, which may not hold true for the complex shapes of marble blocks. Sharp edges and corners introduce discontinuities in conductivity gradients, complicating modeling and potentially leading to erroneous reconstructions. Additionally, imperfections within marble blocks disrupt the continuity of conductivity distributions. Cracks or impurities confound the interpretation of tomographic data, hindering reconstruction accuracy [4]. To adapt the Poisson coefficient to the problem of marble block modulation and characterization, several modifications and considerations need to be made:

- Adjust boundary conditions to account for the complex geometry of marble blocks;
- Modify the Poisson coefficient model to incorporate imperfections such as cracks or impurities within the marble blocks;
- Validate the modified approach experimentally using data obtained from marble block modulation experiments.

By developing improved methods for electric tomography and conductivity modeling in marble blocks, our study aims to innovate and to maintain its competitiveness and foster innovation in the marble sector in Portugal and beyond. Enhanced characterization techniques will enable more efficient extraction, processing, and utilization of marble resources, contributing to sustainable development and economic growth in the sector [5].

Keywords: Electric tomography, Poisson coefficient, marble blocks, conductivity distributions.

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